

ASIAC 49

GD/A 63-3707
ERR-AN-316
Stress

ON NONLINEAR PRESSURE COUPLING IN CYLINDRICAL SHELL ANALYSIS

Paul E. Wilson
Edward E. Spier

1 November 1963

ENGINEERING DEPARTMENT

DISTRIBUTION STATEMENT A
Approved for Public Release
Distribution Unlimited

RETURN TO: AEROSPACE STRUCTURES
INFORMATION AND ANALYSIS CENTER
AFWOL/FBR
WPAFB, OHIO 45433

20000425 154

Reproduced From
Best Available Copy

GD

GENERAL DYNAMICS | ASTRONAUTICS

ON NONLINEAR PRESSURE COUPLING IN CYLINDRICAL SHELL ANALYSIS

Paul E. Wilson
Edward E. Spier

1 November 1963

ENGINEERING DEPARTMENT

This work was supported under General Dynamics sponsored
Research Program Number 111-9582-911

GD

GENERAL DYNAMICS | ASTRONAUTICS

TABLE OF CONTENTS

<u>CONTENTS</u>	<u>PAGE</u>
ILLUSTRATIONS	iii
ABSTRACT	1
1. INTRODUCTION	1
2. NOMENCLATURE	2
3. THEORY	2
4. ANALYSIS	3
5. ACKNOWLEDGMENT	5
6. BIBLIOGRAPHY	6
7. DISTRIBUTION	11

ILLUSTRATIONS

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	Shell Juncture	8
2	Moment Comparison Curves	9
3	Shear Comparison Curves	10

ON NONLINEAR PRESSURE COUPLING IN CYLINDRICAL SHELL ANALYSIS

ABSTRACT

Two well known thin shell theories are used to evaluate and compare shears and moments at the juncture of two pressurized cylindrical shells. For highly pressurized cylinders with large radius-thickness ratios the numerical results indicate that nonlinear coupling effects of pressure significantly influence computed values of discontinuity shears and moments.

1. INTRODUCTION

Many structural problems in the aerospace industry involve highly pressurized shells with large radius-thickness ratios, and under these circumstances, as suggested by Hetenyi (1)¹, the coupling effects of pressure often have a significant influence on discontinuity shears, moments, and stresses. Accordingly, a number of investigations have recently been focused on this problem for pressurized cylindrical (2-8), spherical (9-10), and arbitrary shells of revolution (11). For internally pressurized shells, comparisons made between discontinuity analyses that include and neglect the coupling effects of meridional load (7, 8) imply that use of the refined method generally yields lower calculated maximum stresses and results in a lighter-weight structure.

In this report two well known thin shell theories are used to evaluate and compare shears and moments at the juncture of two pressurized cylindrical shells. For thin highly pressurized cylinders the numerical results indicate that nonlinear coupling effects of pressure have a significant influence on computed values of discontinuity shears and moments.

¹Numbers in parentheses designate references listed in the Bibliography.

2. NOMENCLATURE

a	= radius of shell middle surface
A, B	= constants of integration
D	= flexural rigidity, $Eh^3/12(1-\nu^2)$
E	= modulus of elasticity
h	= shell thickness
i	= $\sqrt{-1}$
k	= shell parameter, Eh/a^2
m	= nondimensional load parameter
M_0, Q_0	= moment and shear at discontinuity neglecting $N_x \frac{d^2 w}{dx^2}$
M_N, Q_N	= moment and shear at discontinuity including $N_x \frac{d^2 w}{dx^2}$
N_x	= axial stress resultant, positive when tensile
p	= internal pressure
w	= radial deflection measured positive inward
w_c, w_p	= complementary and particular solutions, respectively
x	= axial coordinate
$\alpha, \beta, \bar{\beta}, \gamma$	= parameters entering into complementary solution
δ_1, δ_2	= membrane expansions given by Eq. [7]
η	= shell thickness ratio, h_1/h_2
λ^4	= shell parameter, $k/4D$
ν	= Poisson's ratio

3. THEORY

The differential equation that governs small axisymmetric displacements of thin cylindrical shells may, with the notation in the Nomenclature, be written in the form (12)

$$D \frac{d^4 w}{dx^4} - N_x \frac{d^2 w}{dx^2} + \frac{Eh}{a^2} w = -p + \nu \frac{N_x}{a} \quad [1]$$

In many discontinuity analyses (13) it is permissible, as verified by test results (14, 15), to delete the pressure coupling term $N_x \frac{d^2 w}{dx^2}$ from Eq. [1]. However, by parametric evaluation of a simple problem, it will be demonstrated that this simplification process is not generally valid for highly pressurized cylindrical shells with large radius-thickness ratios.

The general solution of Eq. [1] for a semi-infinite cylindrical shell is well known (1) and may be expressed in the form

$$w = w_c + w_p \quad [2]$$

where

$$w_p = - \frac{a^2}{Eh} \left(p - v \frac{N_x}{a} \right) \quad [3]$$

$$N_x < 2\sqrt{kD}, \quad w_c = e^{-\alpha x} (A \sin \beta x + B \cos \beta x)$$

$$N_x = 2\sqrt{kD}, \quad w_c = e^{-\gamma x} (A + Bx) \quad [4]$$

$$N_x > 2\sqrt{kD}, \quad w_c = e^{-\alpha x} (A \sinh \beta x + B \cosh \beta x)$$

and

$$\alpha = \sqrt{\lambda^2 + \frac{N_x}{4D}}, \quad \beta = -i\bar{\beta} = \sqrt{\lambda^2 - \frac{N_x}{4D}}, \quad \gamma = \sqrt{\frac{N_x}{2D}} \quad [5]$$

4. ANALYSIS

Figure 1 shows a longitudinal section of the juncture of two semi-infinite cylindrical shells that are subjected to internal pressure p . Take $h_1 \leq h_2$ and let x_1 and x_2 be directed as shown. With the compatibility and equilibrium conditions at the juncture, along with Eqs. [2-5], it may be shown that the discontinuity moment M_N and shear Q_N act as shown in Fig. 1 and can be expressed in the following form:

$$M_N = \frac{2D_1 D_2 \lambda_1^2 \lambda_2^2 (D_2 \lambda_2^2 - D_1 \lambda_1^2) (\delta_1 - \delta_2)}{(D_1 \lambda_1^2 + D_2 \lambda_2^2)^2 + 2D_1 D_2 \alpha_1 \alpha_2 (\lambda_1^2 + \lambda_2^2) + \frac{N_x}{2} (D_1 \lambda_1^2 + D_2 \lambda_2^2)} \quad [6]$$

$$Q_N = \frac{4D_1 D_2 \lambda_1^2 \lambda_2^2 \left[\alpha_1 \lambda_2^2 D_2 + \alpha_2 \lambda_1^2 D_1 + \frac{N_x}{2} (\alpha_1 + \alpha_2) \right] (\delta_1 - \delta_2)}{(D_1 \lambda_1^2 + D_2 \lambda_2^2)^2 + 2D_1 D_2 \alpha_1 \alpha_2 (\lambda_1^2 + \lambda_2^2) + \frac{N_x}{2} (D_1 \lambda_1^2 + D_2 \lambda_2^2)}$$

where

$$\delta_i = \frac{a^2}{Eh_i} \left(p - \nu \frac{N_x}{a} \right); \quad i = 1, 2 \quad [7]$$

$$N_x = \frac{pa}{2}$$

and the subscript i ($i = 1, 2$) denotes quantities evaluated in the region $0 \leq x_i \leq \infty$. When written in their present form, Eqs. [6] are valid for all values of N_x .

If effects of pressure coupling are neglected the moment M_0 and shear Q_0 at the discontinuity may, by suitable reduction of Eqs. [6], be shown to be

$$M_0 = \frac{2D_1D_2\lambda_1^2\lambda_2^2(D_2\lambda_2^2 - D_1\lambda_1^2)(\delta_1 - \delta_2)}{(D_1\lambda_1^2 + D_2\lambda_2^2)^2 + 2D_1D_2\lambda_1\lambda_2(\lambda_1^2 + \lambda_2^2)} \quad [8]$$

$$Q_0 = \frac{4D_1D_2\lambda_1^3\lambda_2^3(D_1\lambda_1 + D_2\lambda_2)(\delta_1 - \delta_2)}{(D_1\lambda_1^2 + D_2\lambda_2^2)^2 + 2D_1D_2\lambda_1\lambda_2(\lambda_1^2 + \lambda_2^2)}$$

Eqs. [8] agree with results given earlier by Johns (13, 14).

An interesting quantitative comparison of these results may be obtained by forming the dimensionless ratios $\frac{M_N}{M_0}$ and $\frac{Q_0}{Q_N}$ as follows:

$$\frac{M_N}{M_0} = \frac{(1+\eta^2)^2 + 2\eta^{3/2}(1+\eta)}{(1+\eta^2)^2 + 2\eta^{3/2}(1+\eta)\sqrt{(1+m)(1+m\eta^2)} + 2m\eta^2(1+\eta^2)} \quad [9]$$

$$\frac{Q_0}{Q_N} = \frac{1 + \eta^{5/2}}{(1+2m\eta^2)\sqrt{1+m} + \eta^{5/2}(1+2m)\sqrt{1+m\eta^2}} \cdot \frac{M_0}{M_N}$$

where

$$\eta = \frac{h_1}{h_2}, \quad m = \frac{\sqrt{3(1-\nu^2)}}{2} \frac{pa^2}{Eh_1^3} \quad [10]$$

Plots of $\frac{M_N}{M_0}$ and $\frac{Q_0}{Q_N}$ versus η for various values of the nondimensional load parameter m are shown in Figs. 2 and 3, respectively. Note that results obtained including effects of N_x on local bending imply that computed values of the discontinuity bending moment and shear are significantly reduced and

increased, respectively, over corresponding values obtained by neglecting pressure coupling effects. For example, consider the following case:

$$\begin{aligned}a &= 20'' \\h_1 &= 0.10'' \\h_2 &= 0.20'' \\p &= 200 \text{ psi} \\E &= 10 \times 10^6 \text{ psi} \\\nu &= 0.3\end{aligned}$$

Thus

$$\eta = 0.5, \quad m = 0.661$$

and from Figs. 2 and 3

$$M_N = 0.75 M_0, \quad Q_N = 1.39 Q_0$$

Consequently in this instance the discontinuity moments and shears computed by neglecting coupling effects of meridional load are in error by approximately 25% and 39%, respectively.

The stresses have not been discussed in this report. However, preliminary calculations indicate that the maximum stress always occurs in the thinner shell. For small values of η the longitudinal stress governs the design, whereas for large values of η the hoop stress governs. Also, it was found that computed values of the maximum stress are reduced when pressure coupling effects are included in the analysis. In practice it is recommended that the maximum stress be computed by using Eqs. [6] in conjunction with the techniques outlined by Grossman (7) and Smith (8).

5. ACKNOWLEDGMENT

The authors extend their appreciation to Mr. D. R. Cropper and others at General Dynamics/Astronautics for assisting in the preparation of this report.

6. BIBLIOGRAPHY

1. Hetenyi, M., Beams on Elastic Foundation (University of Michigan Press, Ann Arbor, Mich., 1946), 4th ed., Chapt. VI, pp. 127-140.
2. Bijlaard, P. P., "Stresses from radial loads and external moments in cylindrical pressure vessels," *Welding J., Research Supplement*, 608-617, (1955).
3. Short, R. D. and Bart, R., "Analysis for determining stresses in stiffened cylindrical shells near structural discontinuities," *David Taylor Model Basin Report 1065*, (1959).
4. Pulos, J. G. and Salerno, V. L., "Axisymmetric elastic deformations and stresses in a ring-stiffened perfectly circular cylindrical shell under external hydrostatic pressure," *David Taylor Model Basin Report 1497*, (1961).
5. Wei, B. C. F., "Structural analysis of solid propellant rocket casings," *ARS Preprint 1590-61*, (1961).
6. Greenbaum, G. A., "Effect of axial force on influence coefficients for thin cylindrical shells," *M. S. Thesis in Engineering, UCLA*, (1961).
7. Grossman, W. B., "Investigation of maximum stresses in long pressurized cylindrical shells," *AIAA J.* 1, 1129-1132, (1963).
8. Smith, G. W., "Analysis of multiple discontinuities in shells of revolution including coupled effects of meridional load," *General Dynamics/Aeronautics Report No. 63-0044*, 7090 Program No. 3019, (1963).
9. Bijlaard, P. P., "Computation of stresses from local loads in spherical pressure vessels," *Welding Research Council, Bulletin No. 34*, New York, (1957).
10. Cline, G. B., "Effect of pressure - stress coupling upon the influence coefficients of spherical shells," *ASME Winter Annual Meeting*, New York, (1962).
11. Nachbar, W., "Discontinuity stresses in pressurized thin shells of revolution," *Lockheed Missiles and Space Division, IMSD-48483*, (1957).
12. Timoshenko, S. P., Theory of Elastic Stability (McGraw - Hill Book Co. Inc., New York, 1936), Chapt. VII, pp. 423-425.

13. Johns, R. H. and Orange, T. W., "Theoretical elastic stress distributions arising from discontinuities and edge loads in several shell-type structures," NASA TR R-103, (1961).
14. Johns, R. H., Morgan, W. C. and Spera, D. A., "Theoretical and experimental analysis of several typical junctions in space vehicle shell structures," ARS Launch Vehicles: Structures and Materials Conference, Phoenix, Arizona, (1962).
15. Morgan, W. C. and Bizon, P. T., "Experimental investigation of stress distributions near abrupt change in wall thickness in thin-walled pressurized cylinders," NASA TN, to be published.

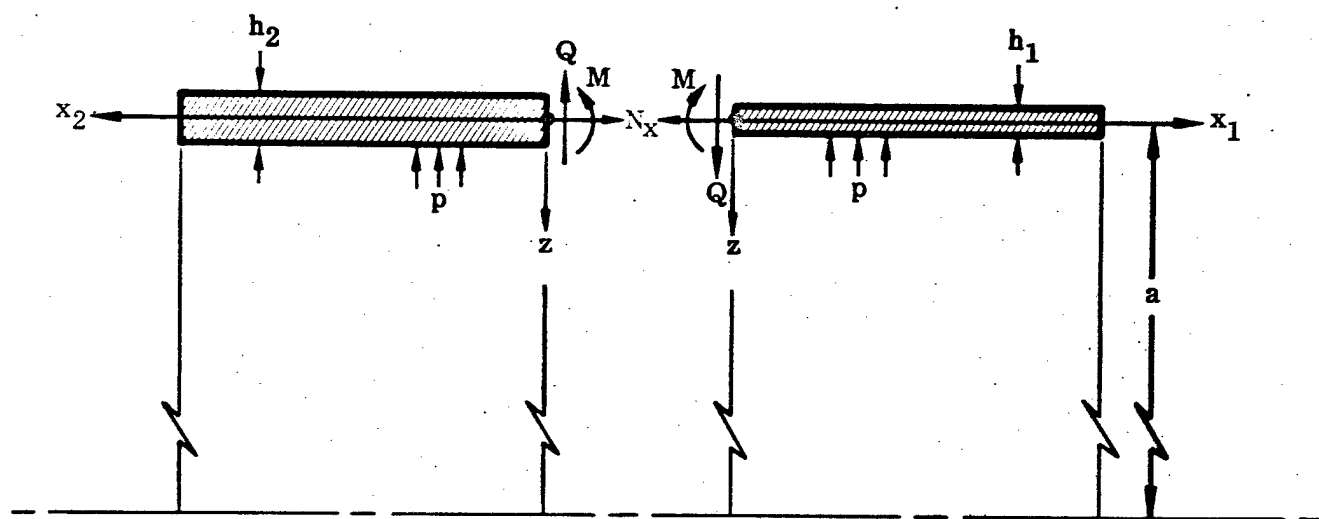


Fig. 1 Shell juncture

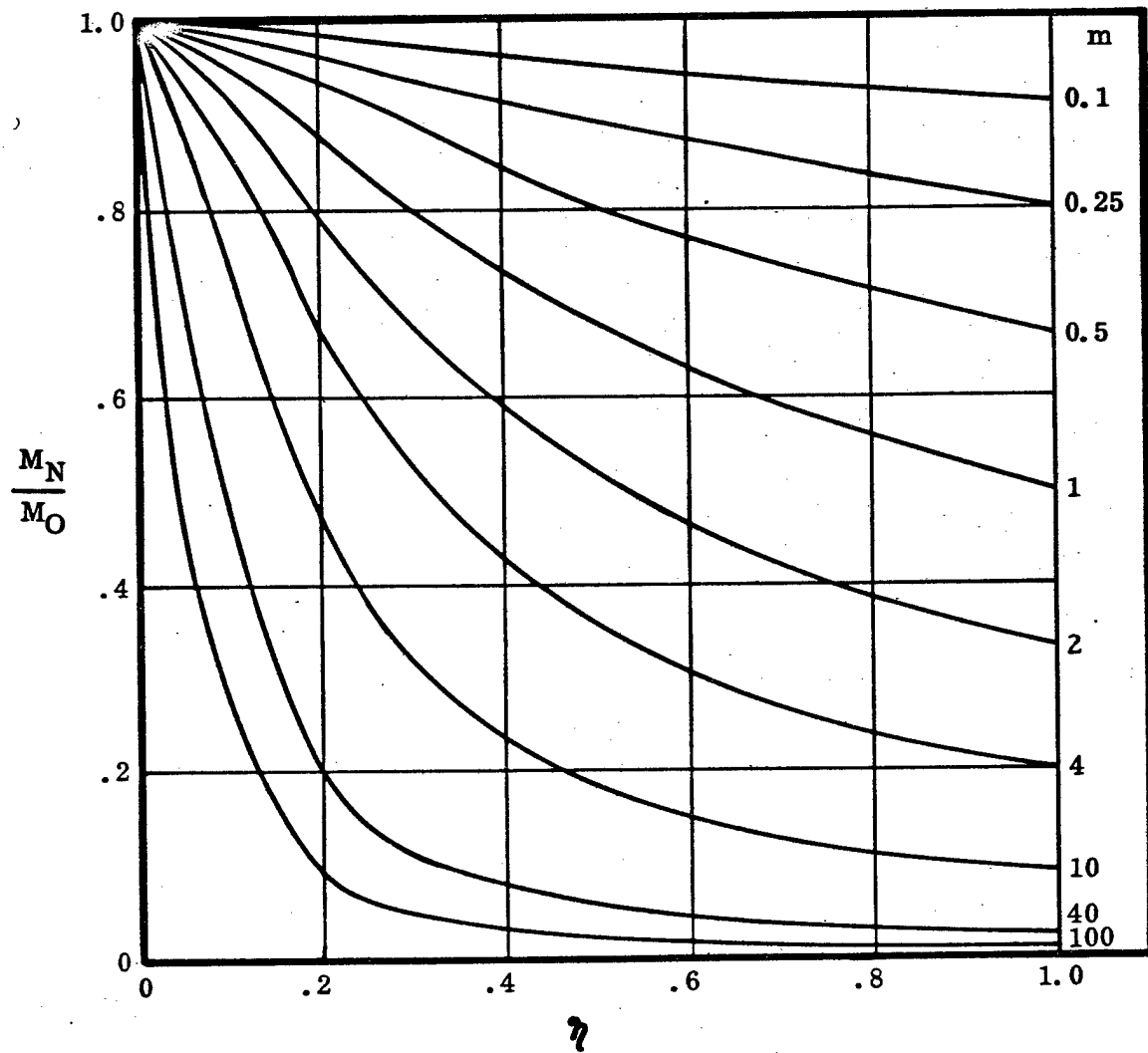


Fig. 2 Moment comparison curves

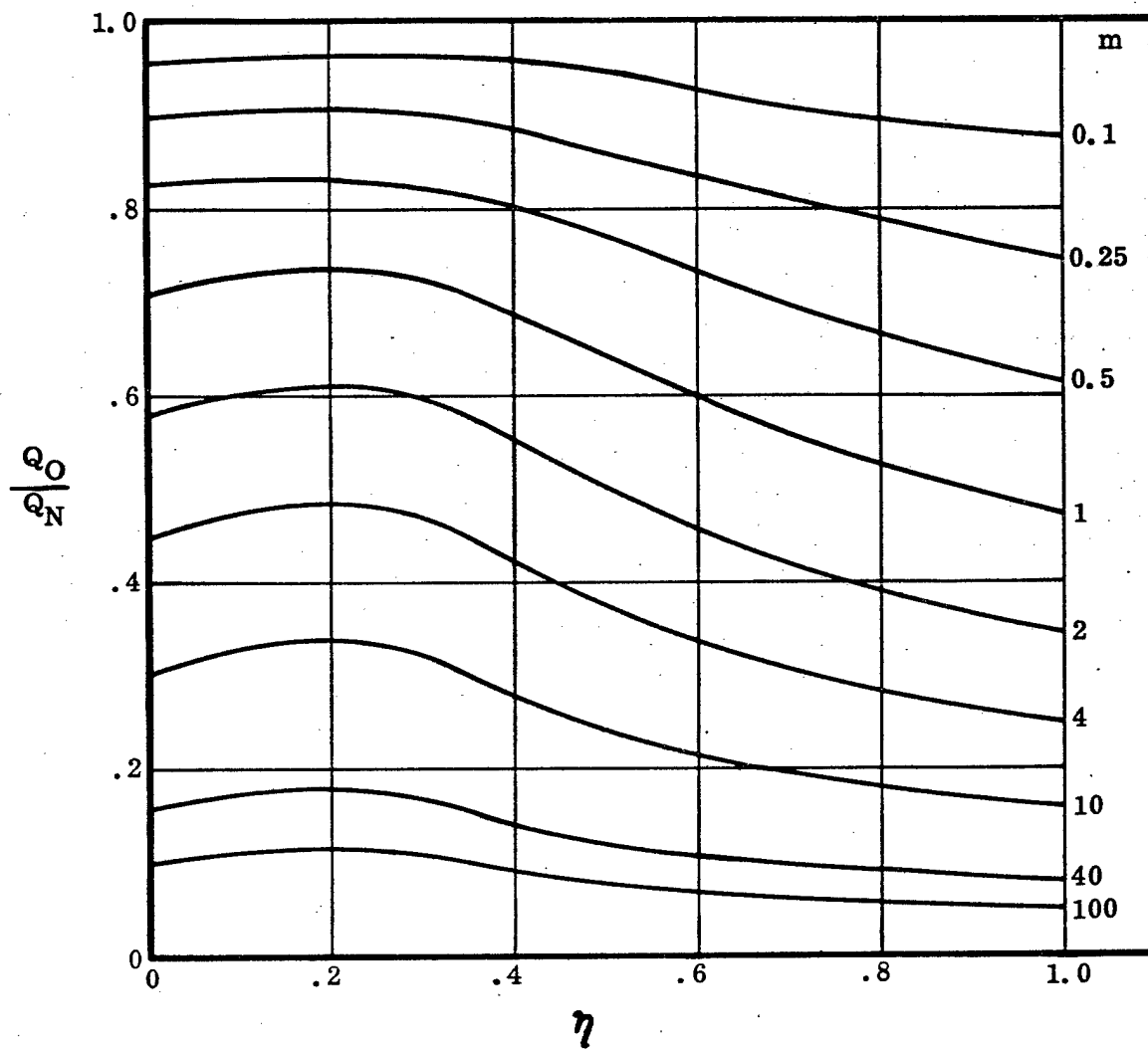


Fig. 3 Shear comparison curves

7. DISTRIBUTION

<u>Division</u>	<u>Recipient</u>	<u>Mail Zone</u>	<u>No. of Copies</u>
General Office (N.Y.)	W. J. Eding	Rockefeller Plaza N.Y. 20, N.Y.	2
Astronautics	H. F. Dunholter	590-00	1
	F. J. Dore	580-00	1
	V. A. Babits	592-00	1
	A. H. Hausrath	557-10	1
	Division Library	128-00	2
Convair	G. G. Green	6-143	2
	Division Library	6-157	1
Fort Worth	E. L. Secrest	E-69	3
	E. H. Watts	T-38	2
	Division Library	E-26	1
Pomona	G. E. Burkheimer	6-1	3
	J. Benish	6-95	1
	M. F. Stoughton	6-95	1
	H. A. Swift	6-56	2
	Division Library	6-20	1

AD NUMBER		DATE	DTIC ACCESSION NOTICE
1. REPORT IDENTIFYING INFORMATION			
A. ORIGINATING AGENCY General Dynamics/Astronautics			
B. REPORT TITLE AND/OR NUMBER GD/A 63-0767 ERR-AN-316			
C. MONITOR REPORT NUMBER			
D. PREPARED UNDER CONTRACT NUMBER			REQUESTER: 1. Put your mailing address on reverse of form. 2. Complete items 1 and 2. 3. Attach form to reports mailed to DTIC. 4. Use unclassified information only. DTIC: 1. Assign AD Number. 2. Return to requester. ASIAC 49
2. DISTRIBUTION STATEMENT A			

DTIC FORM 50
DEC 80

PREVIOUS EDITIONS ARE OBSOLETE

DEFENSE TECHNICAL INFORMATION CENTER

CAMERON STATION
ALEXANDRIA, VIRGINIA 22314

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE, \$300

POSTAGE AND FEES PAID
DEFENSE LOGISTICS AGENCY
DOD-304



AFWAL/FIBRA (ASIAC)
WPAFB, OH 45433
ATTN: CATHERINE WOLF